Modeling of Density Fluctuations in Supersonic Turbulent Boundary Layers

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Nomenclature

c_f = skin friction, $2\tau_w/\rho_e u_e^2$
M = Mach number
r = recovery factor
T = temperature
u = velocity in mean flow direction
y = direction normal to mean flow
δ = boundary-layer thickness
γ = specific heat ratio
ρ = density
τ = turbulent shear stress

Subscripts

e = edge of boundary layer
w = wall

Superscripts

$(\ )'$ = fluctuating quantity
$(\bar{\ })$ = time-averaged quantity

Introduction

THE guidance system for advanced high-speed missiles is a complex missile function. When considering optical system for operation at supersonic or hypersonic conditions, the mean and turbulence density profiles (i.e., the aero-optical effects) must be quantified for the designer. The mean density profiles will cause a shift of the focused spot image relative to the receiver, and the density fluctuations cause the focused image to become blurred. The mean density profiles can be determined from the the solution of the mean conservation equations couples with the equation of state. A method for determining the density fluctuation profile in a compressible, turbulent boundary layer is the subject of this Note.

Previous aero-optical studies have dealt primarily with atmospheric effects on airborne coherent radiation systems.[1] Other applications are reviewed in Ref. 2. This work considered subsonic to transonic compressible boundary-layer effects. Local density fluctuations were determined from thermal anemometry measurements. The flow was assumed adiabatic, and pressure fluctuations were neglected. The previous modeling[3,4] does not determine the local density fluctuations in supersonic boundary layers.

Analysis

The density fluctuation models are derived from the temperature form of the Crocco-Busemann integrals[5] valid for 1) adiabatic flow and 2) zero pressure gradient nonadiabatic flow, respectively,

$$\bar{T}/T_e = 1 + r(\gamma-1)M_e^2[1-(\bar{u}/u_e)^2]/2 \qquad (1)$$

$$\bar{T}/T_e = T_w/T_e - r(\gamma-1)M_e^2[(\bar{u}/u_e - 1/2)^2 - 1/4]/2 - \bar{u}/u_e(T_w/T_e - 1) \qquad (2)$$

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The models are formulated from the differential form of Eqs. (1) and (2). It is assumed that the differential represents the fluctuation of the particular quantity $[d(\ )=(\ )']$. The logarithmic differential form of Eq. (1) is

$$(T'/\bar{T}) = -r(\gamma-1)M^2(u'/\bar{u}) \qquad (3)$$

This relationship without the r term was first reported by Morkovin[6] (see also Refs. 7–9). The Morkovin relation can be derived from the logarithmic differential form of the adiabatic energy equation along an inviscid streamline. Kovasznay[10] also used the logarithmic differential approach for the interpretation of hot-wire measurements in supersonic flow.

The relationship among the fluctuating temperature, pressure, and density is obtained from the equation of state. The pressure fluctuation term must be known in order to evaluate the density fluctuation. Kistler[11] presented plausible arguments for neglecting the pressure fluctuations in supersonic boundary layers, and that assumption will be employed herein. Hence, the density fluctuations are determined from the temperature fluctuations. The resulting local density fluctuation models for 1) adiabatic wall and 2) zero pressure gradient nonadiabatic wall are, respectively,

$$\sqrt{\overline{\rho'^2}}/\bar{\rho} = (c_f/2)^{1/2}(\bar{\rho}\overline{u'^2}/\tau_w)^{1/2} \times [r(\gamma-1)M_e^2](\bar{u}/u_e)/(\bar{T}/T_e)^{1/2} \qquad (4)$$

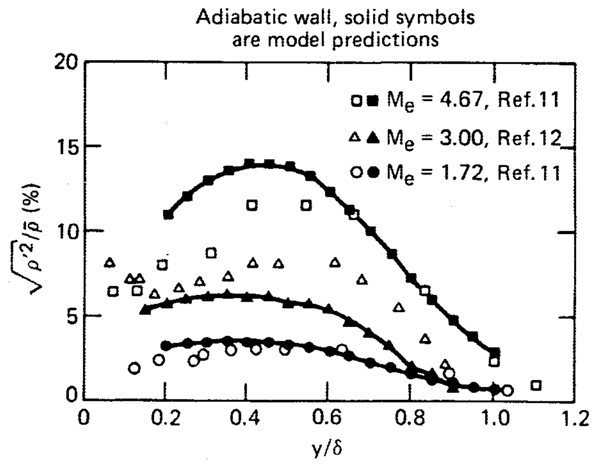

Fig. 1 Comparison between model predictions and experimental data for adiabatic-wall turbulent boundary layers.

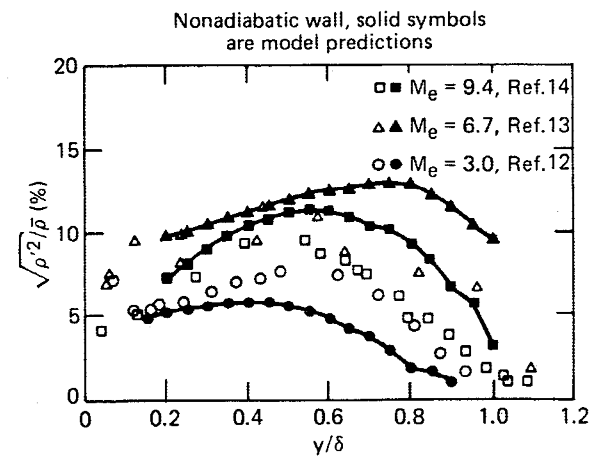

Fig. 2 Comparison between model predictions and experimental data for nonadiabatic-wall turbulent boundary layers.

$$\sqrt{\overline{\rho'^2}}/\bar{\rho} = (c_f/2)^{1/2} (\bar{\rho} \overline{u'^2}/\tau_w)^{1/2} \times [r(\gamma-1)M_e^2(\bar{u}/u_e - 1/2) + (T_w/T_e - 1)]/(\bar{T}/T_e)^{1/2} \tag{5}$$

The temperature distributions in Eqs. (4) and (5) are given by Eqs. (1) and (2), respectively.

Results and Discussion

Sources were located that report experimentally determined local density fluctuation profiles for adiabatic-wall[11,12] and nonadiabatic-wall[12-14] turbulent boundary layers. The reported experimental values for c_f, M_e, T_w/T_e, T_e, and Reynolds normal stress ($\bar{\rho}\overline{u'^2}$) were used in Eqs. (4) and (5). The temperature profiles were computed from a 1/7th-power law velocity profile.

A comparison among Eq. (4) and the experimental data[11,12] for three supersonic boundary layers is shown in Fig. 1. At a fixed y/δ, the model and data increase for increasing M_e. The data and predicted values show a maximum in the midportion of the boundary layer. The model is observed to adequately predict the quantitative behavior in supersonic adiabatic-wall boundary layers.

A comparison among Eq. (5) and the experimental data[12-14] for nonadiabatic boundary layers is shown in Fig. 2. Quantitative agreement among data and model predictions is less favorable than the adiabatic case. The predicted values show a maximum. This trend is in qualitative agreement with the data.

The quantity subject to the greatest computational and measurement uncertainty in the present model is the Reynolds normal stress. The predicted values shown in Figs. 1 and 2 have the same uncertainty as those used in the model calculations. All Reynolds normal stress and density fluctuation data reported herein were obtained from thermal anemometry measurements. It is noted that thermal anemometry measurements can result in erroneously low readings if not properly frequency-compensated.[12] Improper compensation usually results in attenuation of the high-frequency component of the signal. Exact compensation is difficult to achieve since the hot-wire "constants" are functions of the position within the boundary layer. In hypersonic boundary layers, the measured velocity fluctuations are strongly dependent upon the total temperature fluctuations. Total temperature fluctuation measurements can show significant scatter (e.g., see Ref. 13). The aforementioned arguments may explain the large scatter among the nonadiabatic wall data and predicted values shown in Fig. 2. Additional measurements of the Reynolds normal stress and density fluctuations are needed to establish the data trends in the boundary layer.

Conclusions

A model for predicting the local density fluctuations in supersonic turbulent boundary layers is proposed. The model is based upon the differential form of the Crocco-Busemann temperature solutions and negligible pressure fluctuations. Model results were compared against adiabatic-wall, supersonic turbulent boundary-layer data, and showed good agreement. Model results are unable to adequately predict the quantitative behavior of nonadiabatic-wall, turbulent boundary-layer data.

Acknowledgment

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Elimination of Temperature Stratification in a Low-Speed Open-Return Wind Tunnel

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Introduction

AIR in an enclosed room may be stratified, with warm air near the ceiling and cooler air near the floor. In cases where an open-return wind tunnel is operating in a room, this stratification can lead to significant measurement errors, especially when using hot-wire anemometry at low speeds. Even a relatively small temperature stratification near the inlet can be amplified several times in the contraction section of the wind tunnel.

This situation was observed to occur in the low-speed, open-return tunnel shown schematically in Fig. 1. The test section is 300 × 970 mm in cross section and 2.44 mm long, with a free-stream velocity range up to 15 m/s. The room housing the facility is heated by steam radiators. At low tunnel speeds, the air motion induced by the wind tunnel itself is not sufficient to

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